A unitary and causal effective field theory based on the chiral Lagrangian

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Motivation

- Strict ChPT has a limited range of convergence (threshold region).
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Strategy

- 2-channel approximation (πN and γN)$\Longrightarrow$ one is limited by energies $\sqrt{s} \simeq 1300$MeV
- Low energy: tree level amplitude (u and t-channel cuts are taken into account) + one loop to chiral order Q^3
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Partial Wave Dispersion Relation

Unitarity and Analyticity:

$$T_{ab}(\sqrt{s}) = U_{ab}(\sqrt{s}) + \sum_{c,d} \int_{w_{\rm thrs}}^{\infty} \frac{dw}{\pi} \frac{\sqrt{s} - \mu_M}{w - \mu_M} \frac{T_{ac}(w)\, \rho_{cd}(w)\, T^*_{db}(w)}{w - \sqrt{s} - i\epsilon}.$$

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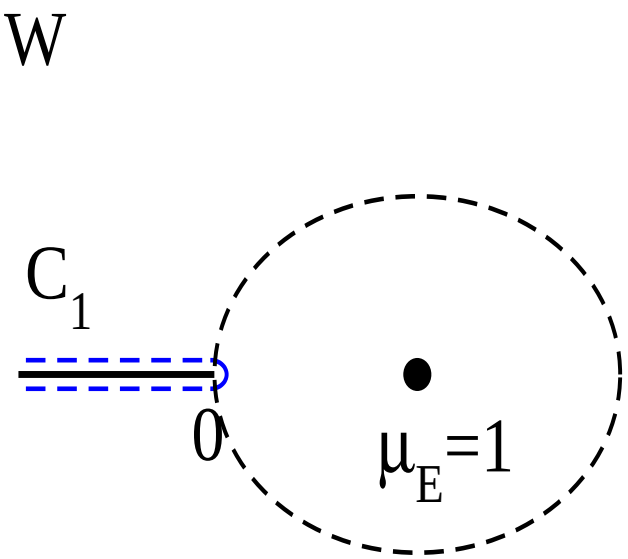
Example of conformal mapping for $U = \log W (W = \sqrt{s})$



$\xi(W) = \frac{1-\sqrt{W}}{1+\sqrt{W}}$ maps the interior of C_1 onto a unit circle.
$\xi(\mu_E) = 0$.

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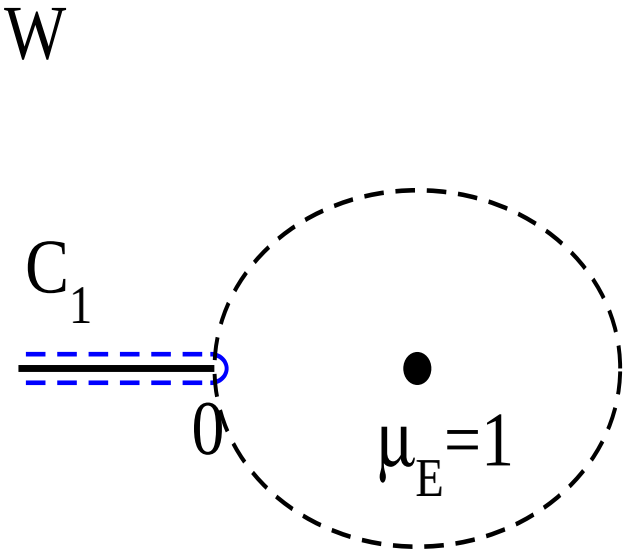

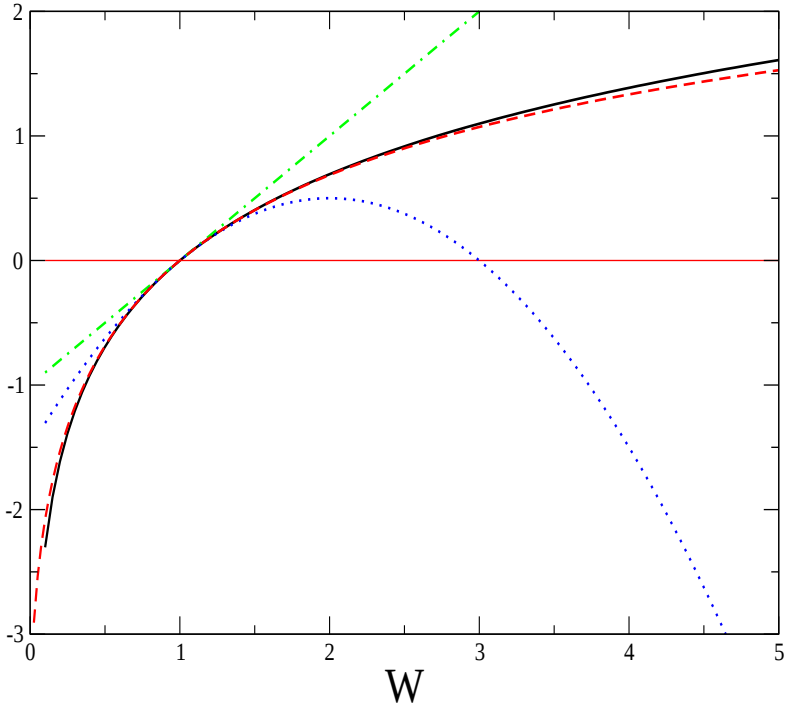

exact

1-st order (Taylor)

2-st order (Taylor)

1-st order (ξ)

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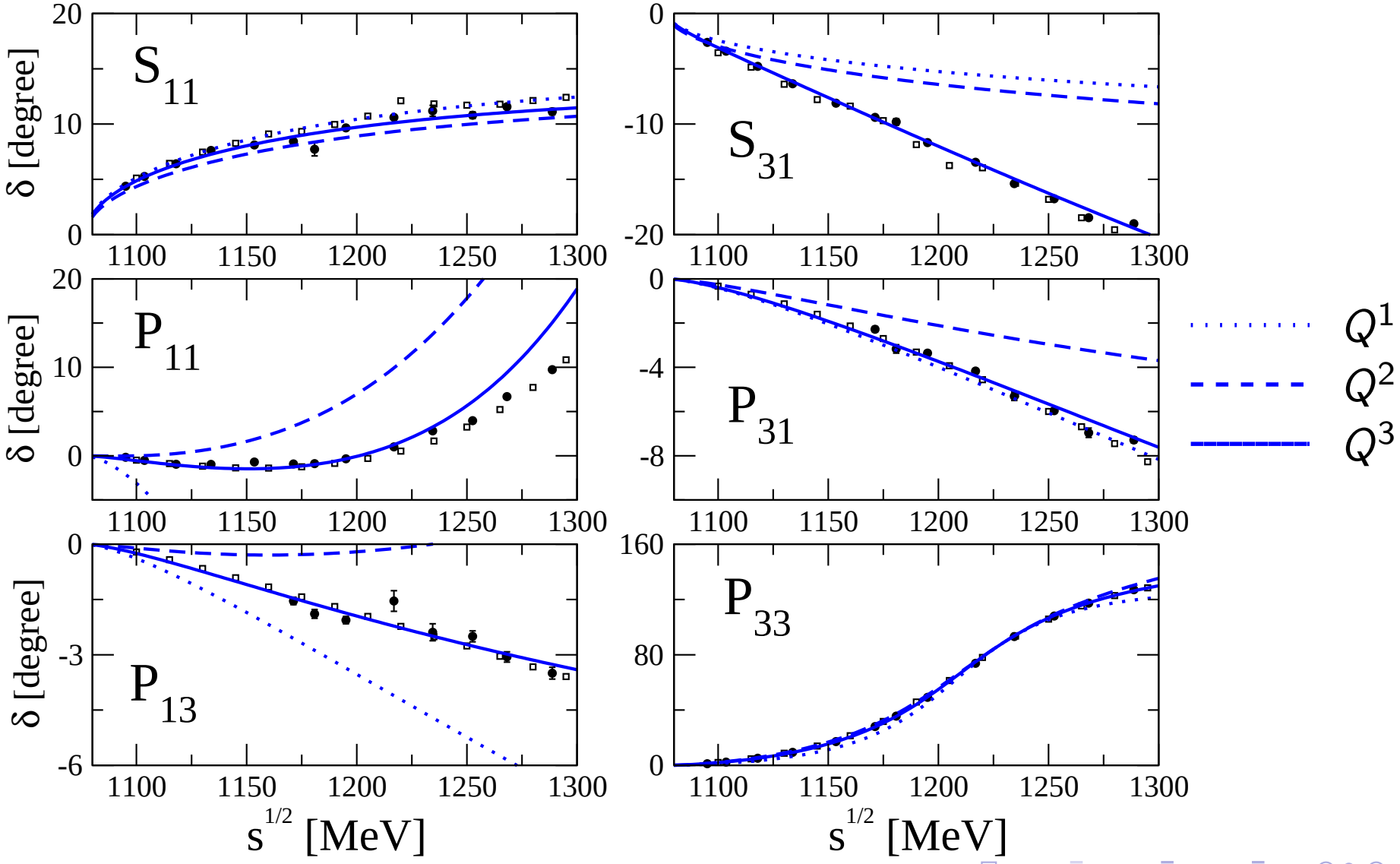
πN phase shifts (S and P waves)



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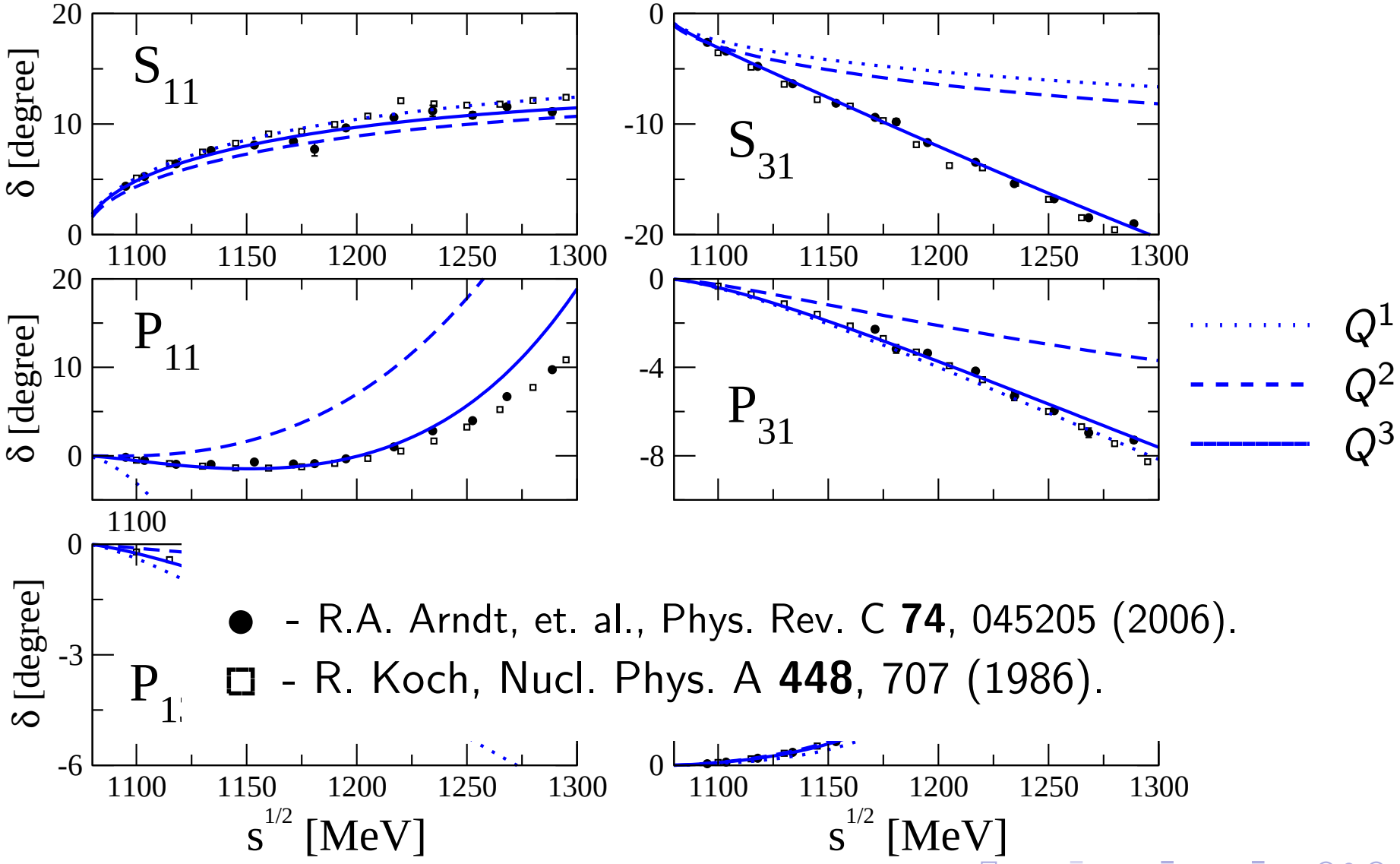

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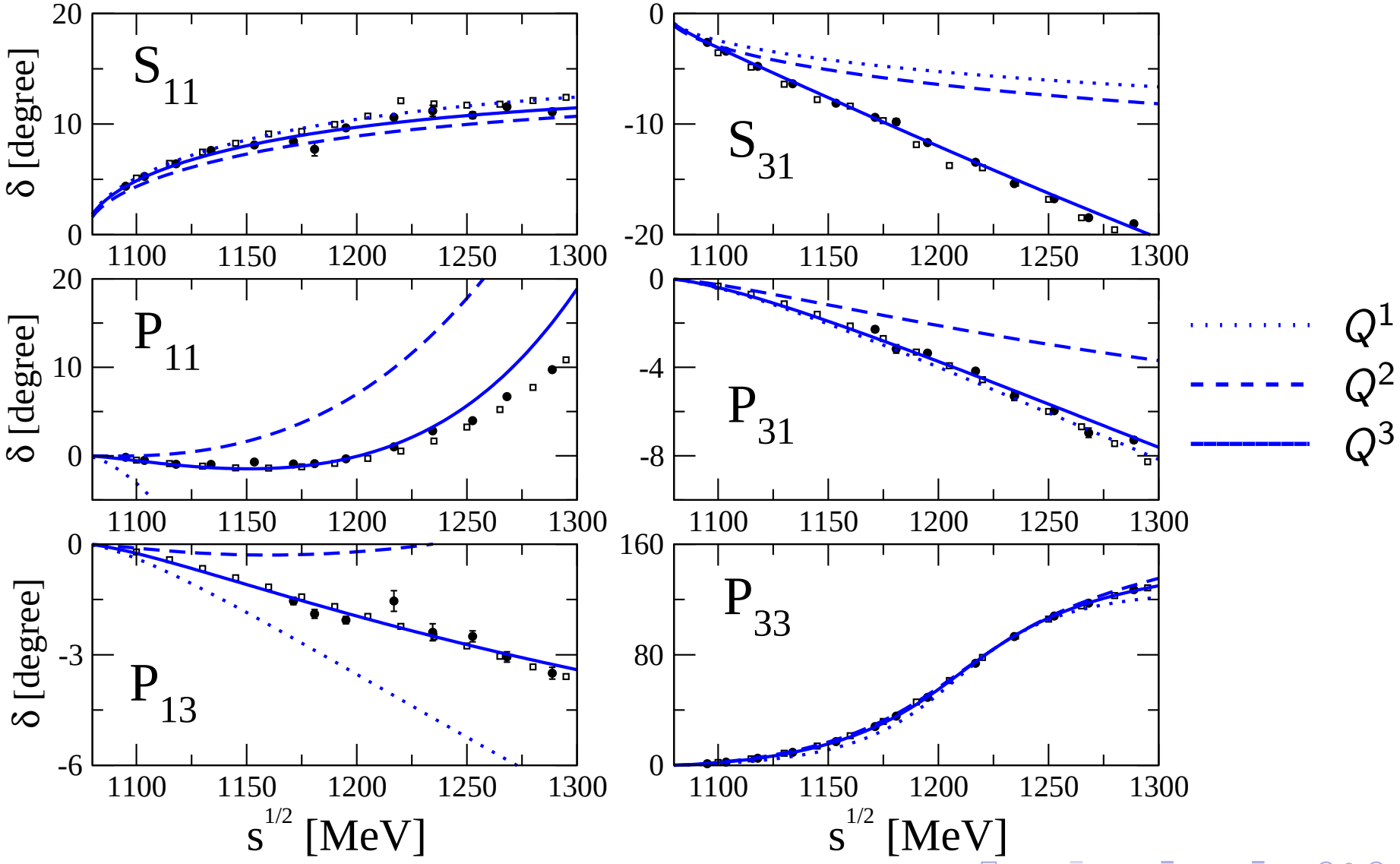


Pion photoproduction

s- and p-waves multipoles are well described up to $\sqrt{s} = 1300$ MeV (at order Q^3).

Threshold data are not included in the fit! (Isospin symmetric case)

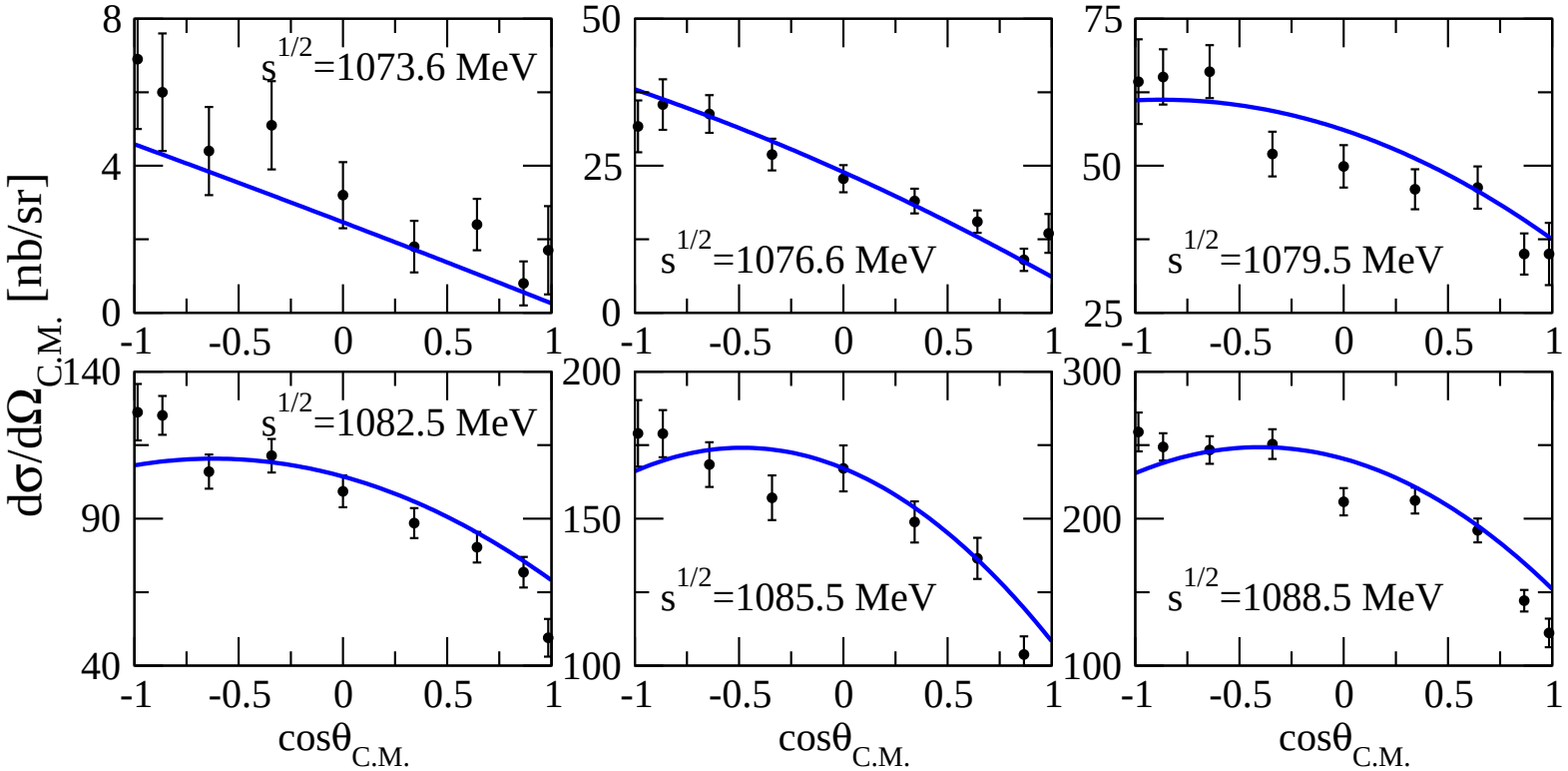
Threshold s-wave multipoles

	Present work	χPT (Q^4)	Experiment
E_{0+} ($\pi^+ n$) [$10^{-3}/m_{\pi^+}$]	27.4	28.2	$28.06 \pm 0.27 \pm 0.45$
E_{0+} ($\pi^- p$) [$10^{-3}/m_{\pi^+}$]	-31.5	-32.7	-31.5 ± 0.8
E_{0+} ($\pi^0 p$) [$10^{-3}/m_{\pi^+}$]	-1.12	-1.16	$-1.32 \pm 0.05 \pm 0.06$ $-1.23 \pm 0.08 \pm 0.03$

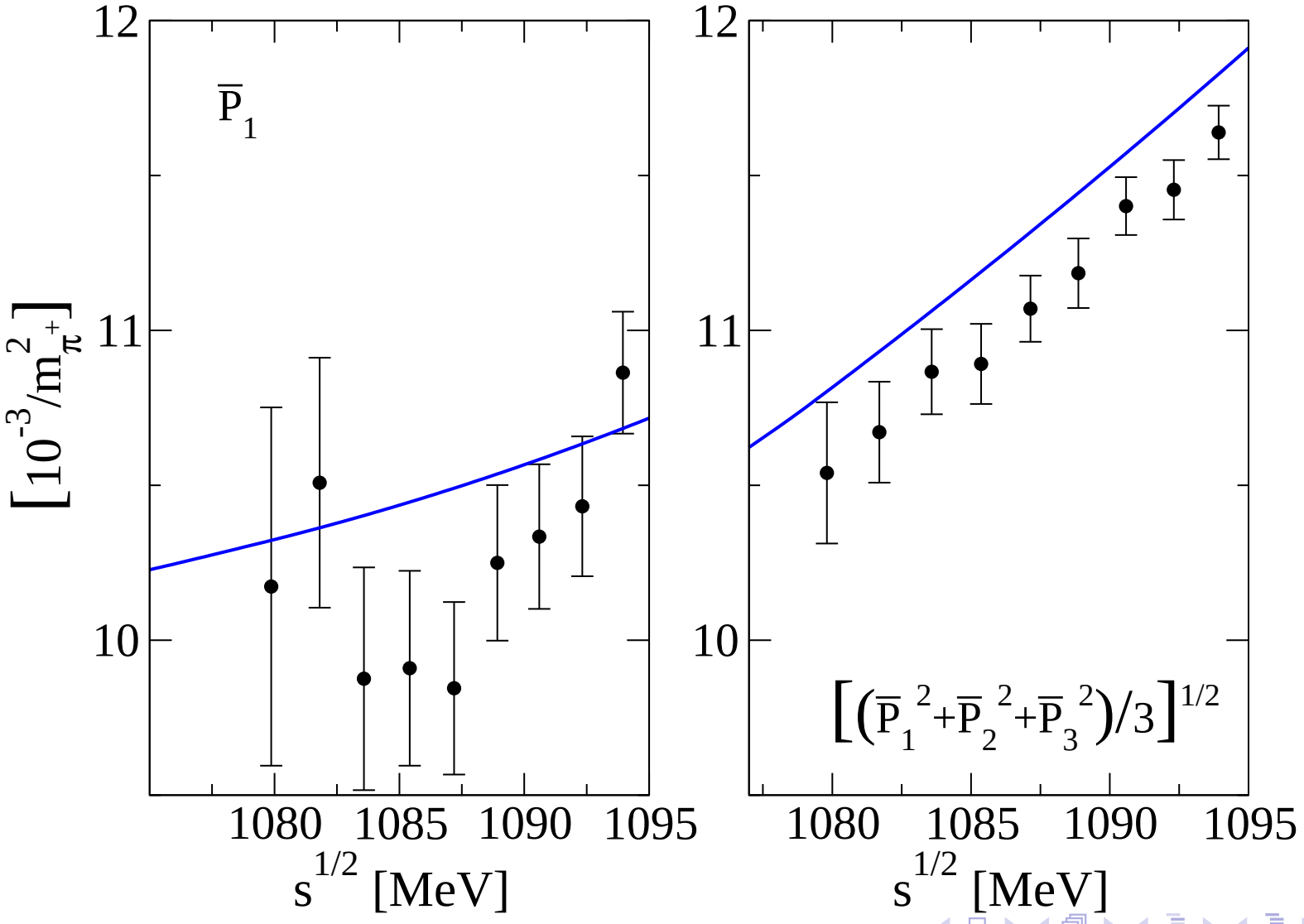
Threshold p-wave multipoles

	Present work	χPT (Q^3)	Experiment
$\bar{P}_1$ ($\pi^0 p$) [$10^{-3}/m^2_{\pi^+}$]	10.2	9.4	$9.46 \pm 0.05 \pm 0.28$
$\bar{P}_2$ ($\pi^0 p$) [$10^{-3}/m^2_{\pi^+}$]	-10.7	-10.0	$-9.5 \pm 0.09 \pm 0.28$
$\bar{P}_3$ ($\pi^0 p$) [$10^{-3}/m^2_{\pi^+}$]	10.3	10.6	$11.32 \pm 0.11 \pm 0.34$

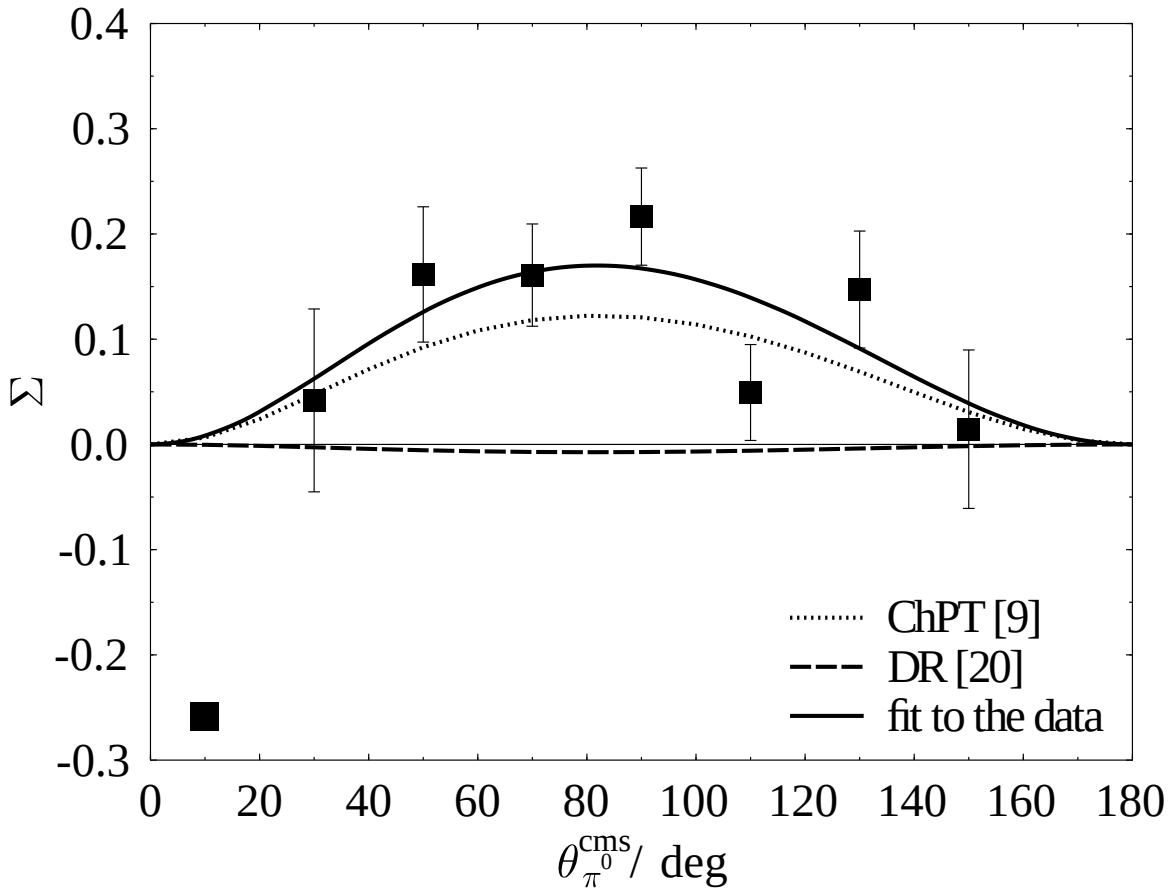
Near threshold differential cross section for the reaction $\gamma p \rightarrow \pi^0 p$ (MAMI 2001)



Energy dependence of the p-wave amplitudes in neutral pion photoproduction (SAS 1996)



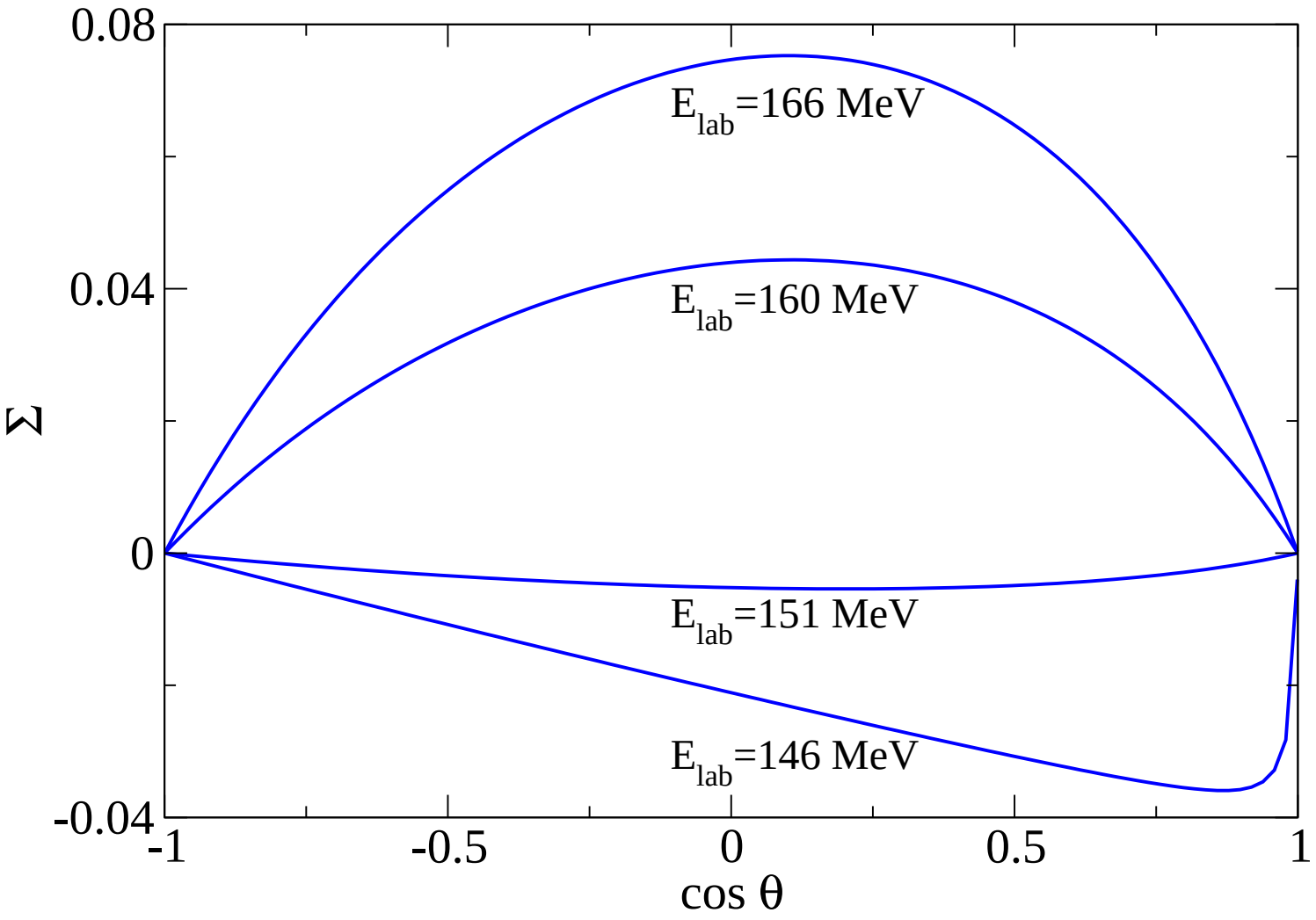
Threshold ($E_\gamma = 159.5$ MeV) photon asymmetry in neutral pion photoproduction (MAMI 2001)



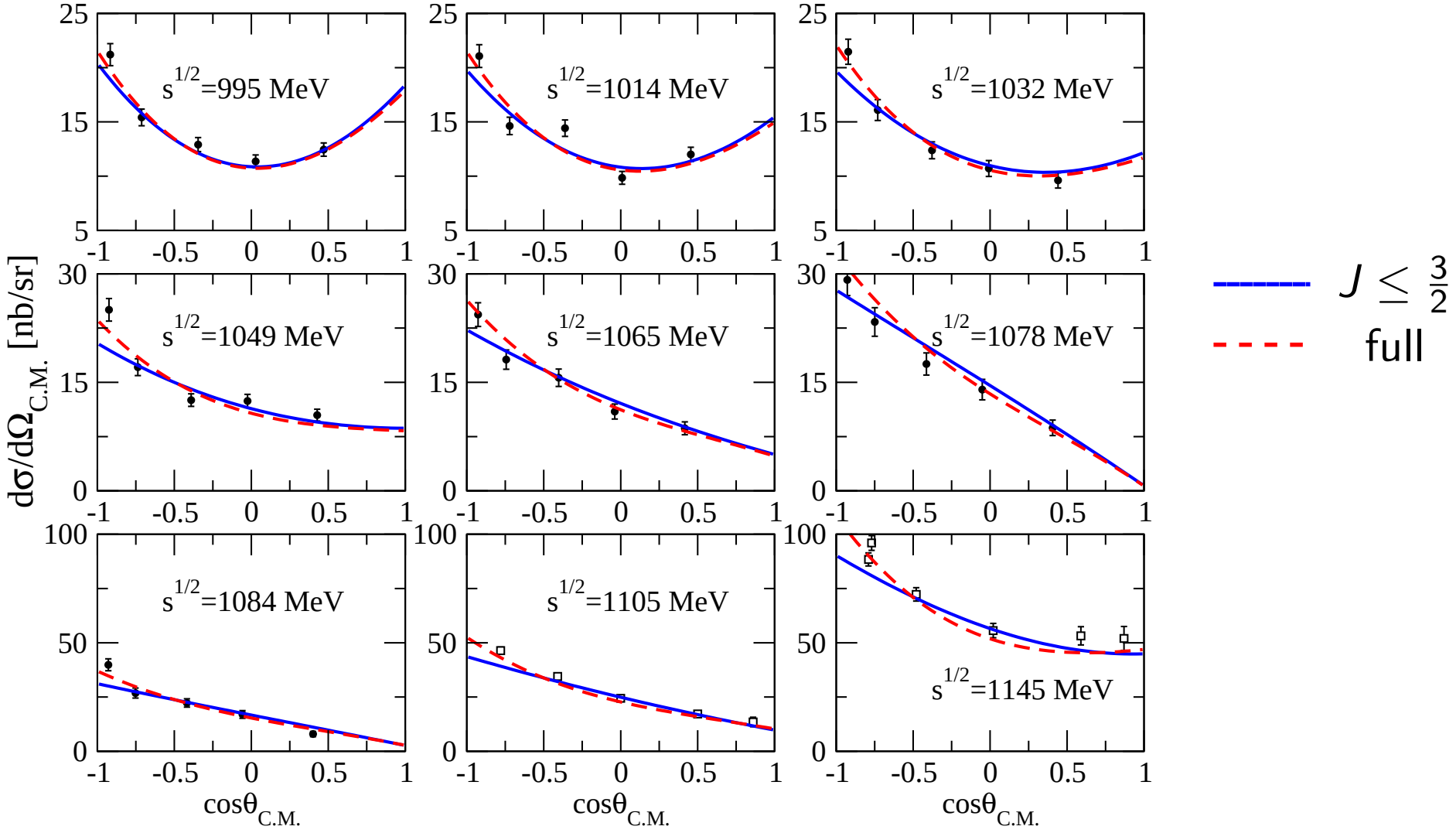
[9]-Bernard et.al. (1996)

[20]-O.Hanstein,D.Drechsel, L.Tiator (1997)

Energy dependence of the photon asymmetry in neutral pion photoproduction



Differential cross section for Compton scattering off the proton



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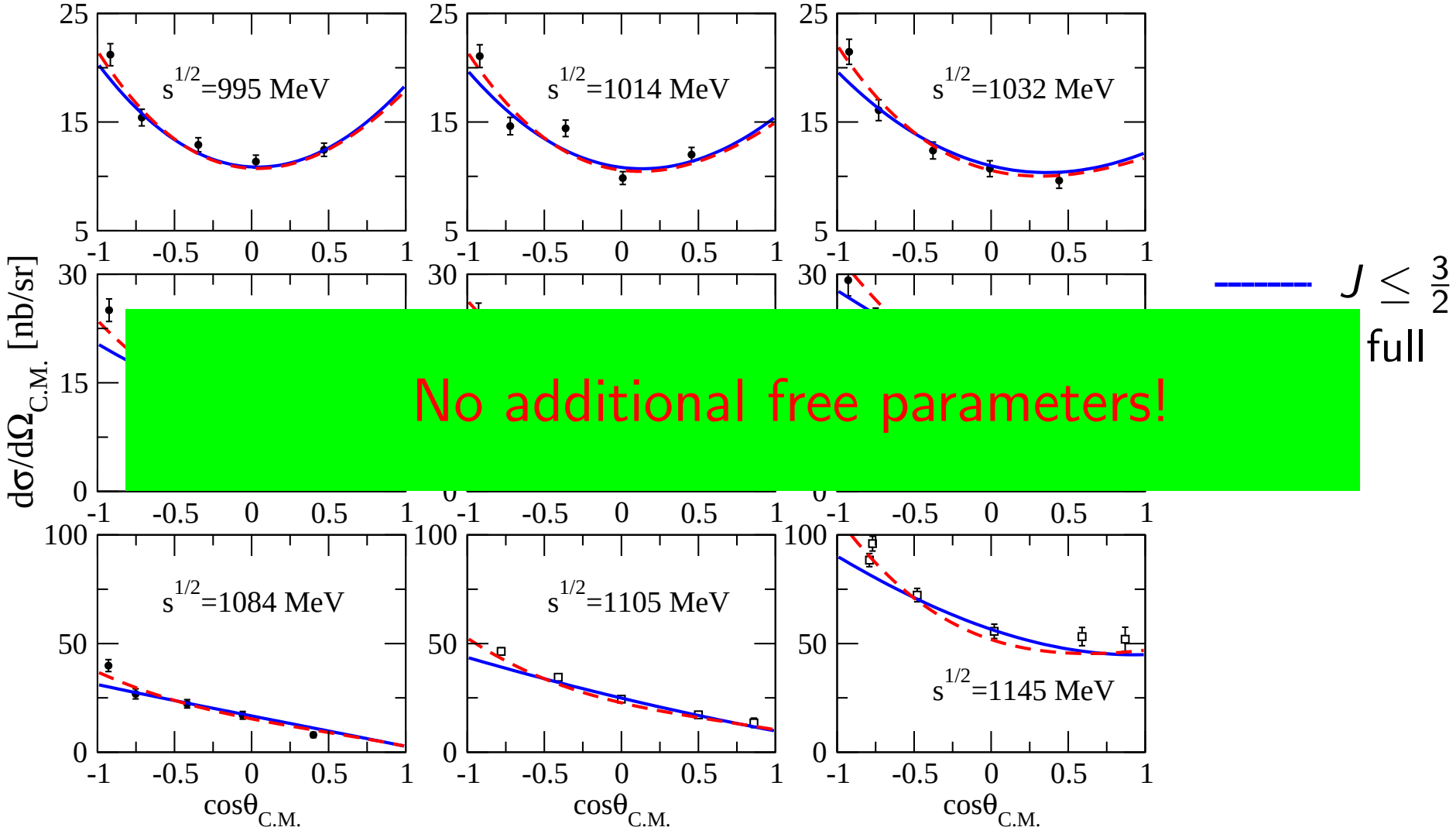

No additional free parameters!

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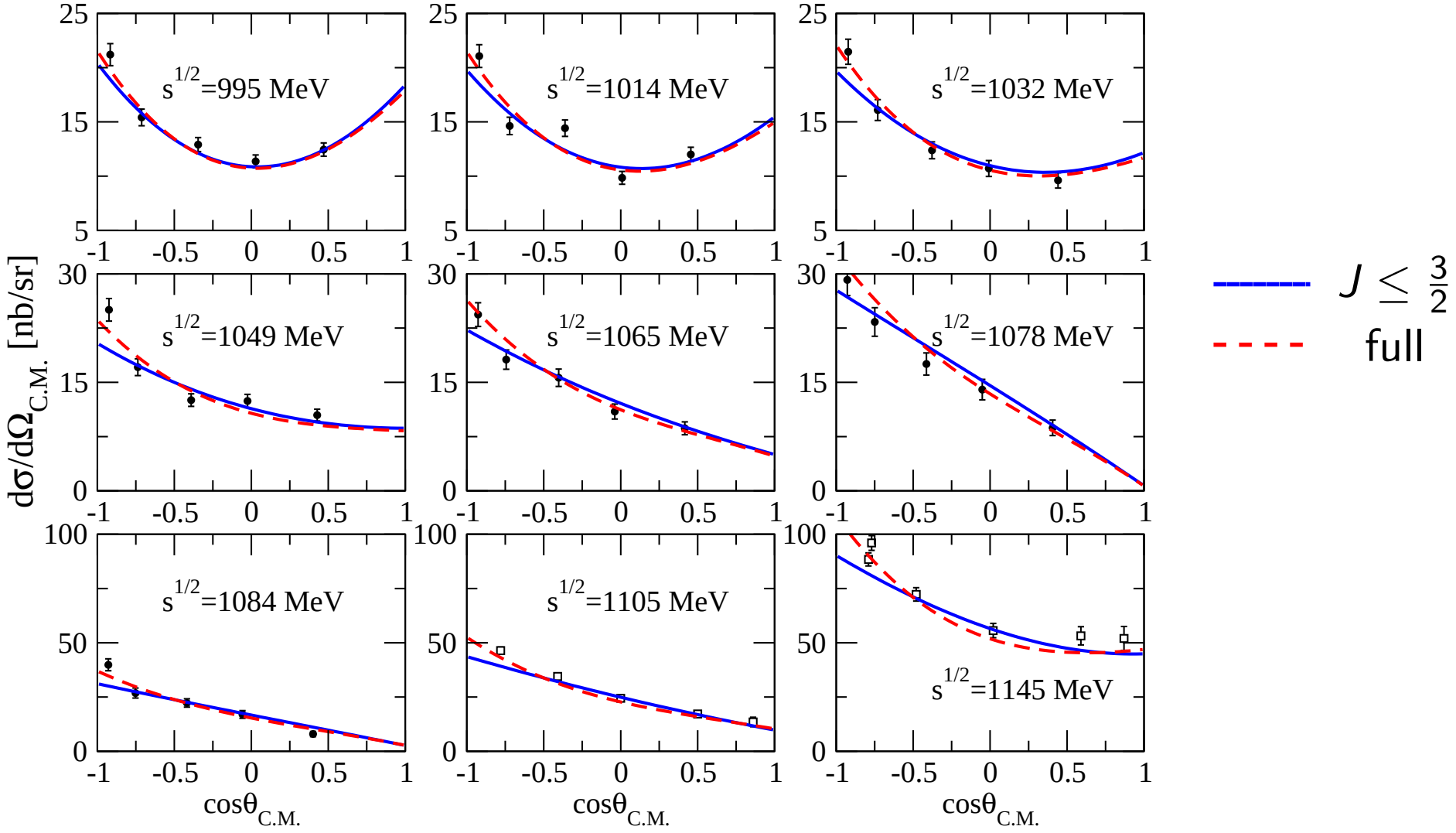

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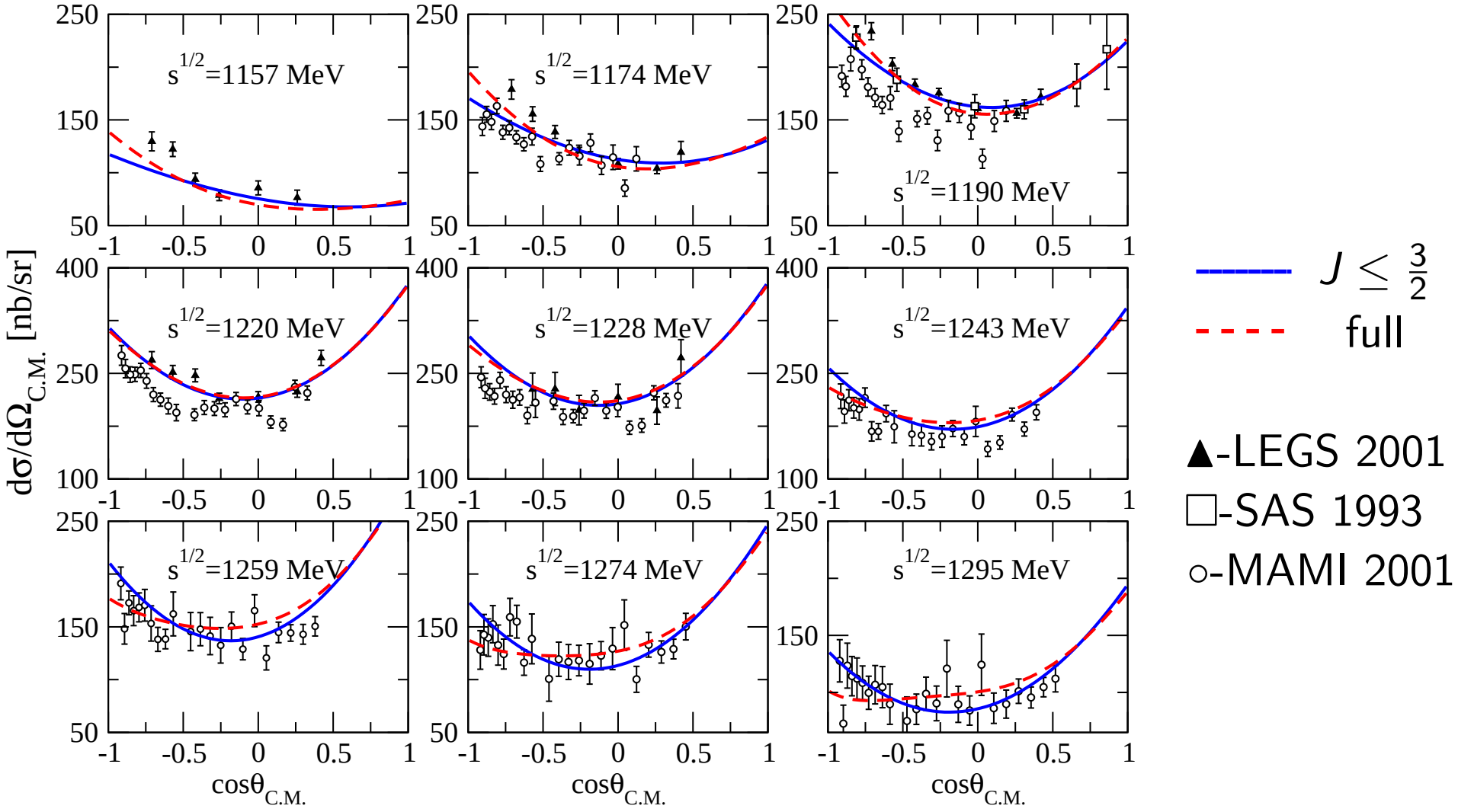
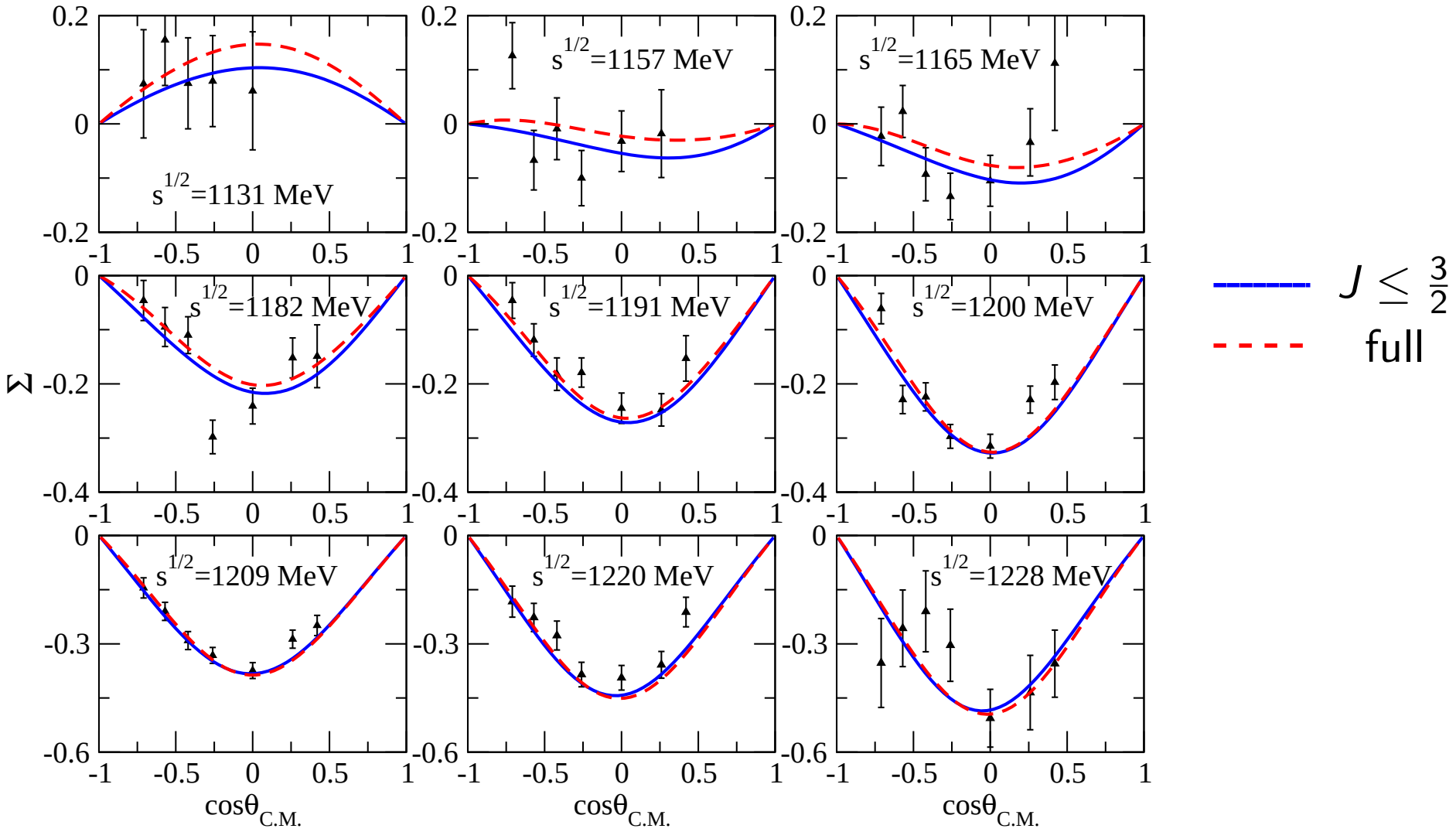


Beam asymmetry for Compton scattering off the proton



Summary

- An application of $SU(2)$ chiral Lagrangian to the order Q^3 is developed.
- Causality and unitarity constraints are utilized to obtain the πN, $\gamma N \to \pi N$ and $\gamma N \to \gamma N$ amplitudes beyond the threshold region.
- With a small number of parameters a good description of πN phase shifts and photoproduction multipoles up to $\sqrt{s} = 1300$ MeV is achieved. Differential cross section and polarization observables for the reactions $\gamma N \to \pi N$ and $\gamma N \to \gamma N$ are also well reproduced.
- Threshold physics is under control. (Parameters are not adjusted to it.)
- To extend the energy region of applicability of the method one needs to include further channels (in progress).